

# Contents

<b>1</b>	<b>Introduction</b>	1
1.1	Problem Definition and Objectives of This Work	1
1.2	Course of Investigation	2
<b>2</b>	<b>Credit Risk Measurement in the Context of Basel II</b>	5
2.1	Banking Supervision and Basel II	5
2.2	Measures of Risk in Credit Portfolios	8
2.2.1	Risk Parameters and Expected Loss	8
2.2.2	Value at Risk, Tail Conditional Expectation, and Expected Shortfall	11
2.2.3	Coherency of Risk Measures	16
2.2.4	Estimation and Statistical Errors of VaR and ES	22
2.3	The Unconditional Probability of Default Within the Asset Value Model of Merton	25
2.4	The Conditional Probability of Default Within the One-Factor Model of Vasicek	28
2.5	Measuring Credit Risk in Homogeneous Portfolios with the Vasicek Model	31
2.6	Measuring Credit Risk in Heterogeneous Portfolios with the ASRF Model of Gordy	35
2.7	Measuring Credit Risk Within the IRB Approach of Basel II	39
2.8	Appendix	43
<b>3</b>	<b>Concentration Risk in Credit Portfolios and Its Treatment Under Basel II</b>	57
3.1	Types of Concentration Risk	57
3.2	Incurrence and Relevance of Concentration Risk	59
3.3	Measurement and Management of Concentration Risk	62

3.4	Heuristic Approaches for the Measurement of Concentration Risk .....	67
3.5	Review of the Literature on Model-Based Approaches of Concentration Risk Measurement .....	70
<b>4</b>	<b>Model-Based Measurement of Name Concentration Risk in Credit Portfolios .....</b>	<b>73</b>
4.1	Fundamentals and Research Questions on Name Concentration Risk .....	73
4.2	Measurement of Name Concentration Using the Risk Measure Value at Risk .....	75
4.2.1	Considering Name Concentration with the Granularity Adjustment .....	75
4.2.2	Numerical Analysis of the VaR-Based Granularity Adjustment .....	87
4.3	Measurement of Name Concentration Using the Risk Measure Expected Shortfall .....	103
4.3.1	Adjusting for Coherency by Parameterization of the Confidence Level .....	103
4.3.2	Considering Name Concentration with the Granularity Adjustment .....	108
4.3.3	Moment Matching Procedure for Stochastic LGDs .....	114
4.3.4	Numerical Analysis of the ES-Based Granularity Adjustment .....	121
4.4	Interim Result .....	134
4.5	Appendix .....	136
<b>5</b>	<b>Model-Based Measurement of Sector Concentration Risk in Credit Portfolios .....</b>	<b>183</b>
5.1	Fundamentals and Research Questions on Sector Concentration Risk .....	183
5.2	Incorporation of Sector Concentrations Using Multi-Factor Models .....	185
5.2.1	Structure of Multi-Factor Models and Basel II-Consistent Parameterization Through a Correlation Matching Procedure .....	185
5.2.2	Accounting for Sector Concentrations with the Model of Pykhtin .....	190
5.2.3	Accounting for Sector Concentrations with the Model of Cespedes, Herrero, Kreinin and Rosen .....	197
5.2.4	Accounting for Sector Concentrations with the Model of Düllmann .....	202

- 5.3 Performance of Multi-Factor Models ..... 212
  - 5.3.1 Analysis for Deterministic Portfolios ..... 212
  - 5.3.2 Simulation Study for Homogeneous and Heterogeneous Portfolios ..... 215
- 5.4 Interim Result ..... 219
- 5.5 Appendix ..... 220
  
- 6 Conclusion** ..... 237
  
- References** ..... 241

# List of Figures

Fig. 2.1	Probability mass function of portfolio losses for an exemplary portfolio .....	15
Fig. 2.2	Limiting loss distribution of Vasicek (1991) .....	34
Fig. 3.1	Types of concentration risk .....	58
Fig. 3.2	Accuracy of the Pillar 1 capital requirements considering risk concentrations .....	66
Fig. 3.3	Lorenz curve for credit exposures .....	68
Fig. 4.1	Value at risk for a wide range of probabilities .....	88
Fig. 4.2	Value at risk for high confidence levels .....	89
Fig. 4.3	Granularity add-on for heterogeneous portfolios calculated analytically with first-order ( <i>solid lines</i> ) and second-order ( <i>dotted lines</i> ) adjustments as well as with Monte Carlo simulations (+ and o) using three million trials .....	102
Fig. 4.4	Value at risk in the ASRF and the Vasicek model .....	104
Fig. 4.5	Different value at risk measures in the Vasicek model .....	106
Fig. 4.6	Expected shortfall in the ASRF and the Vasicek model .....	107
Fig. 4.7	Portfolio quality distributions .....	109
Fig. 4.8	Probability distribution of recovery rates for corporate bonds and loans, 1970–2003 .....	115
Fig. 4.9	Expected shortfall for a wide range of probabilities .....	122
Fig. 4.10	Expected shortfall for high confidence levels .....	122
Fig. 4.11	ES-based granularity add-on for heterogeneous portfolios calculated analytically with first-order ( <i>solid lines</i> ) and second-order ( <i>dotted lines</i> ) adjustments as well as with Monte Carlo simulations (+ and o) using three million trials .....	133
Fig. 4.12	Relation between the shift of the probability and the loss quantile .....	143

Fig. 5.1 Diversification Factor realizations on the basis  
of 50,000 simulations ..... 200

Fig. 5.2 Surface plot of the  $DF$ -function ..... 201

Fig. 5.3 Deviations of  $VaR^{Basel}$  and  $VaR^{mf}$  from  $ES^{mf}$  ..... 218

# List of Tables

Table 2.1	Loss distribution for an exemplary portfolio .....	14
Table 3.1	Guidance for institutions and supervisors considering concentration risk .....	64
Table 4.1	Critical number of credits from that ASRF solution can be stated to be sufficient for measuring the true VaR (see (4.49)) ....	92
Table 4.2	Critical number of credits from that the exact solution at confidence level 0.995 exceeds the infinite fine granularity at confidence level 0.999 (see (4.50)) .....	93
Table 4.3	Critical number of credits from that the first order adjustment can be stated to be sufficient for measuring the true VaR (see (4.51)) .....	96
Table 4.4	Critical number of credits from that the first order adjustment at confidence level 0.995 exceeds the infinite fine granularity at confidence level 0.999 (see (4.52)) .....	97
Table 4.5	Critical number of credits from that the first plus second order adjustment can be stated to be sufficient for measuring the true VaR (see (4.53)) .....	99
Table 4.6	Critical number of credits from that the first plus second order adjustment at confidence level 0.995 exceeds the infinite fine granularity at confidence level 0.999 (see (4.54)) .....	101
Table 4.7	Confidence level for the ES so that the ES is matched with the VaR with confidence level 0.999 for portfolios of different quality .....	108
Table 4.8	Recovery rates by seniority, 1970–2003 .....	119
Table 4.9	Results of the normal distribution .....	120
Table 4.10	Results of the lognormal distribution .....	120
Table 4.11	Results of the logit-normal distribution .....	120
Table 4.12	Results of the beta distribution .....	120

Table 4.13	Critical number of credits from that ASRF solution can be stated to be sufficient for measuring the true ES if LGDs are deterministic (see (4.98)) .....	125
Table 4.14	Critical number of credits from that ASRF solution can be stated to be sufficient for measuring the true ES if LGDs are stochastic (see (4.99)) .....	126
Table 4.15	Critical number of credits from that the first order adjustment can be stated to be sufficient for measuring the true ES if LGDs are deterministic (see (4.100)) .....	127
Table 4.16	Critical number of credits from that the first order adjustment can be stated to be sufficient for measuring the true ES if LGDs are stochastic (see (4.101)) .....	129
Table 4.17	Critical number of credits from that the first plus second order adjustment can be stated to be sufficient for measuring the true ES if LGDs are deterministic (see (4.102)) .....	131
Table 4.18	Critical number of credits from that the first plus second order adjustment can be stated to be sufficient for measuring the true ES if LGDs are stochastic (see (4.103)) .....	132
Table 5.1	Inter-sector correlation structure based on MSCI industry indices (in %) .....	188
Table 5.2	Overall sector composition of the German banking system .....	189
Table 5.3	Implicit intrasector correlations for different portfolio qualities .....	189
Table 5.4	Parameter combinations for the calibration of the model .....	210
Table 5.5	Comparison of the models for the five benchmark portfolios with absolute error in basis points (bp) and relative error in percent (%) .....	213
Table 5.6	Comparison of the models for five high concentrated portfolios with absolute error in basis points (bp) and relative error in percent (%) .....	214
Table 5.7	Comparison of the models for five low concentrated portfolios with absolute error in basis points (bp) and relative error in percent (%) .....	215
Table 5.8	Accuracy of different models in comparison with the “true” ES calculated with Monte Carlo simulations for the specified simulation studies .....	216
Table 5.9	Comparison of the runtime .....	218